Computably Enumerable Sets in the Solovay and the Strong Weak Truth Table Degrees

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Abstract. The strong weak truth table reducibility was suggested by Downey, Hirschfeldt, and LaForte as a measure of relative randomness, alternative to the Solovay reducibility. It also occurs naturally in proofs in classical computability theory as well as in the recent work of Soare, Nabutovsky and Weinberger on applications of computability to differential geometry. Yu and Ding showed that the relevant degree structure restricted to the c.e. reals has no greatest element, and asked for maximal elements. We answer this question for the case of c.e. sets. Using a doubly non-uniform argument we show that there are no maximal elements in the sw degrees of the c.e. sets. We note that the same holds for the Solovay degrees of c.e. sets.

1 Introduction

The strong weak truth table reducibility was suggested by Downey, Hirsch feldt, and LaForte as a measure of relative randomness. Versions of this reducibility are present in computability theory; for instance, these are automatically produced by the basic technique of 'simple permitting' and one of them was used in the recent work of Soare, Nabutovsky and Weinberger on applications of computability theory to differential geometry. The strong weak truth table reducibility naturally induces a degree structure, the sw degrees. Yu and Ding showed that the sw degrees restricted to the c.e. reals have no greatest element, and asked for maximal elements. We solve this question for the case of c.e. sets. Using a doubly non-uniform argument we show that there are no maximal elements in the sw degrees of the c.e. sets. The strong weak truth table reducibility was originally suggested as an alternative for the Solovay (or *domination*) reducibility which has been very successful tool for the study the complexity of c.e. reals but presents various shortcomings outside this class. Of course, the sw degrees present other difficulties (as the lack of join operator, see below) but they are nevertheless very interesting to study from a wider perspective. Moreover, Downey, Hirschfeldt and LaForte [2] noticed that as far as the computably enumerable sets are concerned, the sw degrees *coincide* with the Solovay degrees. So we also show that the Solovay degrees of c.e. sets have no maximal element.

In the following we assume basic computability theory background; knowledge of algorithmic randomness is not essential but can be useful. For definitions, motivation and history of related notions as the Solovay degrees we refer mainly to [1] and secondly to [4].

Studying relative randomness, Downey, Hirschfeldt and LaForte [2] found Solovay reducibility insufficient, especially as far as non-c.e. reals are concerned. One of the two new measures for relative randomness they suggested is a strengthening of the weak truth table reducibility, which they called *strong weak truth table reducibility* or sw for short. This reducibility is quite natural since it occurs in many proofs in classical computability theory: it follows when we apply simple permitting for the construction of a set 'below' a given one.

Definition 1. *(Downey, Hirschfeldt and LaForte [2]) We say $A \leq_{sw} B$ if there is a Turing functional Γ and a constant c such that $\Gamma^B = A$ and the use of this computation on any argument n is bounded by $n + c$.*

The special case when $c = 0$ gives a stronger reducibility which was used by Soare, Nabutovsky and Weinberger (see [7]) in applying computability theory to differential geometry.

We remind the definition of a c.e. real.

Definition 2. *A real number is* computably enumerable (c.e.) *if it is the limit of a computable increasing sequence of rationals.*

The main justification for $\leq_{\mathrm{sw}}$ as a measure of relative randomness was the following

Proposition 1. *(Downey, Hirschfeldt, LaForte [2]) If $a \leq_{sw} b$ are c.e. reals then for all n, the prefix-free complexity of $a \upharpoonright n$ is less than or equal to that of $b \upharpoonright n$ (plus a constant).*

So $\leq_{\mathrm{sw}}$ arguably qualifies as a *measure of relative randomness* for the c.e. reals (and in particular, it preserves randomness). Downey, Hirschfeldt, LaForte [2] have showed that Solovay reducibility (also known as *domination*) and strong weak truth table reducibility *coincide* on the c.e. sets. But, as we see below, this is not true for the c.e. reals.

Yu and Ding proved the following

Theorem 1. *(Yu and Ding [6]) There is no sw-complete c.e. real.*

By a 'uniformization' of their proof they got two c.e. reals which have no c.e. real sw-above them. Hence

Corollary 1. *(Downey, Hirschfeldt, LaForte [2])*
The structure of sw-degrees is not an upper semi-lattice.

They also asked whether there are maximal sw-degrees of c.e. reals. They conjectured that there are such, and they are exactly the ones that contain random c.e. reals. The main idea of their proof of theorem 1 can be applied for the case of c.e. sets in order to get an analogous result. Using different ideas we prove the following stronger result.

Theorem 2. *There are no sw-maximal c.e. sets. That is, for every c.e. set A, there exists a c.e. set W such that $A <_{sw} W$.*

Since the Solovay degrees and sw-degrees coincide on the c.e. sets we get

Corollary 2. *The substructure of the Solovay degrees consisting of the ones with c.e. members (i.e. containing c.e. sets) has no maximal elements.*

2 About the structure

We state some easy results about the c.e. sets and reals in the structure of sw degrees. We remind the following definition:

Definition 3. *A (total) Solovay test is a c.e. set of binary strings S such that $\sum_{\sigma\in S} 2^{-|\sigma|} < \infty$ (and computable). A real a avoids S if*

$$\exists^{<\infty}\sigma \in S(\sigma \sqsubset a).$$

(Schnorr) Random is a real which avoids all (total) Solovay tests.

After the discussion in the previous section, it is natural to ask: are there c.e. reals above all c.e. sets?

Proposition 2. *Every random c.e. real is sw-above every set in the finite levels of the difference hierarchy.*

But are there non-random c.e. reals with this property?

Proposition 3. *There are non-random c.e. reals sw-above every set in the finite levels of the difference hierarchy.*

E.g. $a = \sum_{e\in\mathbb{N}} \sum_{n\in W_e} 2^{-(e+n+2)}$ is non-random and sw-above all c.e. sets.

Question 1. Are the c.e. reals above the c.e. sets necessarily Schnorr random?

3 About the proof of theorem 2

Before we get into the technical part of the proof, we draw a map of it. Given a c.e. set A we construct three c.e. sets W_1, W_2, W_3 *one of which* will be strictly sw-above A. Figure 1 illustrates this idea and shows the double non-uniform nature of the proof, which will become more clear in the technical part (in particular, W_1 will be *qualitatively different* from W_2, W_3). We note that some of W_i may not be able to even compute A. Given a c.e. set A we construct a c.e. W which satisfy the requirements

$$\mathcal{Q} : A \leq_{\mathrm{sw}} W$$
$$\mathcal{N}_e : \Phi_e^A \neq W$$

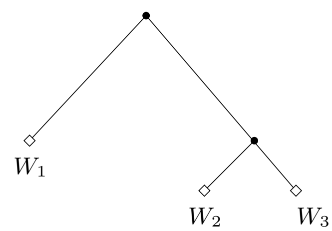

Fig. 1. Double non-uniformity in the proof of theorem 2

where Φ_e is the e-th sw-functional in an effective enumeration and has use (bounded by) $n+e$ on argument n. Although $\mathcal{N}_e$ can be satisfied with a non-uniform proof in the style of [6] (thus showing that there is no sw-maximum c.e. set), adding $\mathcal{Q}$ makes the situation more difficult. In particular, for any number of W-witnesses that we reserve for $\mathcal{N}_e$, we need to reserve roughly the same number of W-witnesses for A-coding (i.e. $\mathcal{Q}$), and these must be roughly of the same level of magnitude as those for $\mathcal{N}_e$. But this is impossible since once you have occupied an interval of $\mathbb{N}$ for $\mathcal{N}_e$ (as in [6]) you can't always find another equally big (disjoint) interval of numbers not much larger than the ones in the first interval. Note that building a large (finite) number of (candidates for) W doesn't help much since each of these W will have the need of *double space* discussed above.

To illustrate the above, consider an attempt to diagonalize against $\Phi_0^A = W$ with a witness w. W.l.o.g. assume that the constant associated with our reduction $A \leq_{\text{sw}} W$ is 0 (i.e. the use is the identity function). We wait until $\Phi_0^A(w)\downarrow = 0$ and put $w \searrow W$. Then $A \upharpoonright w$ may change (in order to rectify the computation) and this change must be coded in W *below* w. So one diagonalization requires two witnesses.

To deal with this situation we will require A to be 'sufficiently charged' in the sense that the 1s in its characteristic sequence are sufficiently dense. Relying on this assumption, we won't enumerate an axiom for the reduction $A \leq_{\text{sw}} W$ unless we witness a certain amount of enumeration in A. This way we save positions in W which would have to be used for the A-coding, had we not waited for this enumeration to occur. If our hypothesis is true, the construction will build one W which fulfils the requirements.

Of course, this 'density' hypothesis (which is the base of *case A* of the proof) is not without loss of generality. A second construction (*case B*) will assume the failure of the density hypothesis, and produce two sets W_1, W_2; if indeed the hypothesis fails, one of these will satisfy all the requirements. Overall we construct three different sets and so this proof is non-uniform.

Case *A.* The hypothesis is

$$\forall e, c \exists \ell (|\overline{A} \upharpoonright (\ell + e)| + |\overline{A} \cap (c, \ell]| < \ell - c \tag{1}$$

and the instance of it used by $\mathcal{N}_e$ is

$$\exists \ell (|\overline{A} \upharpoonright (\ell + e)| + |\overline{A} \cap (c, \ell]| < \ell - c \tag{2}$$

where c is the largest number reserved for diagonalization and A-coding by the higher priority requirements $\mathcal{N}_i, i < e$. Every requirement reserves a full interval of $\mathbb{N}$ for enumeration into W and $\mathcal{N}_e$ in particular reserves $(c, \ell]$, where ℓ is the least witness for (2). So, in this case $(-1, c]$ is full of numbers reserved by $\mathcal{N}_i, i < e$. Every time a requirement reserves an interval $(c, \ell]$, it automatically starts sw-coding $A \cap (c, \ell]$ into $W \cap (c, \ell]$ (the use function of the reduction being the identity). This means that from now on every time that a new element n appears in $A \cap (c, \ell]$, we enumerate a number $t \leq n$ into $W \cap (c, \ell]$. In this setting, condition 2 guarantees that although we will need to spend part of $(c, \ell]$ for A-coding, there will still be enough witnesses for a successful ripple of $\mathcal{N}_e$-diagonalizations (i.e. one that finishes with a diagonalization which is not rectified).

Since the largest number reserved by $\mathcal{N}_e$ is the ℓ of 2, if c_e, ℓ_e are the c, ℓ of condition 2 for $\mathcal{N}_e$ then $c_e = \ell_{e-1}$ where $\ell_{-1} = -1$. It is easy to see that a list

$$\ell_0 < \ell_1 < \ell_2 < \dots$$

of suitable endpoints for all the requirements can be effectively obtained by choosing an ℓ_e to be one of the ℓ of 2 for $c = \ell_{e-1}$ (say the first that occurs during the given enumeration of A). So we divide $\mathbb{N}$ into

$$(-1, \ell_0], (\ell_0, \ell_1], \dots,$$

sw-code $A \cap (\ell_{e-1}, \ell_e]$ into $W \cap (\ell_{e-1}\ell_e]$ for each e, and use the rest of the witnesses for a diagonalization ripple for $\mathcal{N}_e$ (i.e. a sequence of diagonalizations where each of them is performed after the previous one has been rectified). It is straightforward to use an initial segment of $(\ell_{e-1}, \ell_e]$ for the A coding and the rest of it for diagonalizations. So we injectively map (the current value of) $\overline{A} \cap (\ell_{e-1}, \ell_e]$ *onto* an initial segment of $(\ell_{e-1}, \ell_e]$ in an *order-preserving* way and for the sake of $\mathcal{Q}$ require that whenever an element of that $\overline{A} \cap (\ell_{e-1}, \ell_e]$ appears in A, the corresponding element (which belongs to the same interval) is enumerated into W. It is obvious that each element of $(\ell_{e-1}, \ell_e]$ is mapped to a number less than or equal to itself and so the coding is sw with the identity as use function.

$\mathcal{N}_e$*-Module.*

1. *(Set up)* Wait until ℓ_{e-1} has been defined and there is an $\ell > \ell_{e-1}$ as in 2 with $c = \ell_{e-1}$. Define $\ell_e = \ell$ and the attack interval

$$I_e = (\ell_{e-1}, \ell_e].$$

   Injectively map $I_e \cap \overline{A}$ onto an initial segment of I_e in an order-preserving way (this can be done in a unique way).

2. *(Diagonalization)* Wait until $\ell(\Phi_e^A, W) > \ell_e$ and put $\max(\overline{W} \cap I_e) \searrow W$.

Each of these strategies require attention when they are ready to move on to the next step (note that part 2 is a loop). Step 1 is performed only once for each requirement and so there will be no injury.

$\mathcal{Q}$-Module. Let Γ be the functional we build for the reduction $A \leq_{\text{sw}} W$.

1. *(Γ rectification)* Search Γ up to the finite current level of its definition and find the least n with
$$\Gamma^W(n) \downarrow \neq A(n).$$
Then $n \in (\ell_i, \ell_{i+1}]$ for some i; enumerate into W the corresponding element of n under the injective mapping defined during the definition of ℓ_{i+1}.
2. *(Γ enumeration)* Let i be the largest number such that $\ell_i \downarrow$. If $\Gamma^W(\ell_i) \uparrow$ then enumerate axioms $\Gamma^W = A$ up to ℓ_i with use function the identity.

Construction. At each stage:

– Run $\mathcal{Q}$-module
– Run $\mathcal{N}_e$-module for the highest $\mathcal{N}$ requiring attention.

Verification. By induction we show that for every e, $\mathcal{N}_e$ is satisfied and Γ is defined and correct up to ℓ_e. Since by 1 ℓ_e is eventually defined for all e and $\ell_e < \ell_{e+1}$, this is all we need to show. Supposing that it holds for all $i < e$, we show that it is true for e. Suppose that ℓ_e is defined at stage s_0. At this stage $W \cap (\ell_{e-1}, \ell_e]$ is empty and according to the $\mathcal{Q}$-module the only numbers in $(\ell_{e-1}, \ell_e]$ enumerated into W by this strategy will be because of numbers appearing in $A \cap (\ell_{e-1}, \ell_e]$ *after* stage s_0.

By the first step of the strategy $\mathcal{N}_e$, at s_0 we have

$$|\overline{A} \upharpoonright (\ell_e + e)| + |\overline{A} \cap (\ell_{e-1}, \ell_e]| < |(\ell_{e-1}, \ell_e]|. \tag{3}$$

Strategy $\mathcal{Q}$ can enumerate into $W \cap (\ell_{e-1}, \ell_e]$ no more than the first $|\overline{A}[s_0] \cap (\ell_{e-1}, \ell_e]|$ elements of $(\ell_{e-1}, \ell_e]$. Also, no other strategy apart from $\mathcal{N}_e$ can enumerate numbers of this interval into W. So according to 3 there will be more than $|\overline{A}[s_0] \upharpoonright (\ell_e + e)|$ for the use of $\mathcal{N}_e$. Each time the agreement $\Phi_e^A = W$ exceeds ℓ_e, this strategy will perform a diagonalization. After each diagonalization, the length of agreement can only exceed ℓ_e again if a number enters A below $\ell_e + e$. Hence there will be a diagonalization which cannot be rectified and this shows that $\mathcal{N}_e$ succeeds.

On the other hand, since $\mathcal{N}_e$ chooses as diagonalization witness the largest element of $(\ell_{e-1}, \ell_e]$ not yet in W, it follows from 3 that the first $|\overline{A}[s_0] \cap (\ell_{e-1}, \ell_e]|$ elements will not be used by this strategy (since, by the time it would need to use them it will have reached a diagonalization which cannot be rectified). And of course they are not going to be used by other $\mathcal{N}$ strategies nor by $\mathcal{Q}$ for the sake of numbers appearing in A outside $(\ell_{e-1}, \ell_e]$. So any of these will stay outside W until (if ever) its corresponding element (under the injective mapping defined in step 1 of $\mathcal{N}_e$, which is greater or equal to it) enters A. So $\mathcal{Q}$ will always be able to rectify (and refresh) Γ on $(\ell_{e-1}, \ell_e]$. So, using the induction hypothesis, eventually Γ will be defined and correct up to ℓ_e. This concludes the induction step and the verification. Note that the reduction of A to W just described is also a many-one reduction.

Case B**.** Suppose that 1 does not hold and so

$$\exists e, c \forall \ell (|\overline{A} \upharpoonright (\ell + e)| + |\overline{A} \cap (c, \ell]| > \ell - c)$$

which can be written as

$$\exists e, c \forall \ell > e, (|\overline{A} \upharpoonright \ell| + |\overline{A} \cap (c, \ell - e]| > \ell - e - c)$$

which implies

$$\exists c \forall \ell > c, (2|\overline{A} \upharpoonright \ell| > \ell - c).$$

But the latter can be written as

$$\exists c \forall \ell > c, (2|A \upharpoonright \ell| < \ell + c).$$

If there is some $0 \leq c_1 < c$ such that

$$\exists^\infty \ell > c, (2|A \upharpoonright \ell| \geq \ell + c_1)$$

there will be a greatest such. For that one it would be

$$\exists^\infty \ell > c, (2|A \upharpoonright \ell| = \ell + c_1)$$

for a possibly different constant c. But then A would be computable which is a trivial case. So we may assume that there is no such c_1 and hence

$$\exists c \forall \ell > c, (2|A \upharpoonright \ell| < \ell).$$

By finitely modifying A (e.g. set it empty up to c) we get

$$\forall \ell > 0, (2|A \upharpoonright \ell| < \ell). \tag{4}$$

This extra hypothesis does not restrict the result, since if it holds for a set then it holds for any finite modification of it. Now 1 allows us to sw-code A into W by only using the even numbers. So we reserve $2\mathbb{N}$ for $\mathcal{Q}$ and define the coding as follows.

$\mathcal{Q}$-Module. If some n has just been enumerated into A, put the largest even number $\leq n$ of $\overline{W}$ into W.

By this procedure for any n, $A \upharpoonright n$ is coded in the even positions of $W \upharpoonright n$. We may fix an enumeration of A in which at most one number appears in it at each stage. To see that the coding succeeds we prove the following

Lemma 1. *If $W(2k) = 0$ at some stage, then*

$$|A \upharpoonright 2k| \geq |\{2t < 2k \mid W(2t) = 1\}|$$

at the same stage.

Proof of lemma Indeed, each even number in $W \upharpoonright 2k$ must have been the code of some number n enumerated in A in a previous stage. But no such n (i.e. one that triggered enumeration into $W \upharpoonright 2k$) can be $\geq 2k$ since $\mathcal{Q}$ would have chosen $2k$ or greater (since $W(2k) = 0$ such codes were available). □

Now the coding works unless there is a stage s where some $n \searrow A$ and all even numbers $\leq n$ are already in W. If $2k$ is the least even not yet in W, $2k > n \geq 0$ and lemma 1 implies that

$$|A \upharpoonright 2k| \geq k$$

holds at this stage, which contradicts 4. So $\mathcal{Q}$ indeed makes sure that $A \leq_{\text{sw}} W$. Of course this is independent with what we do with the odd numbers in relation to W, which we are going to use for satisfying the $\mathcal{N}$ requirements.

We will construct two sets W_1, W_2 both sw-above A (via the $\mathcal{Q}$-strategy) one of which will satisfy all $\mathcal{N}$ requirements. So we can replace $\mathcal{N}$ by

$$\mathcal{N}'_e : \Phi_e^A \neq W_1 \ \vee \ \Psi_e^A \neq W_2.$$

Each $\mathcal{N}'_e$ will occupy the odd numbers of an interval $[2c_e + 1, 2c_{e+1} + 1)$ of $\mathbb{N}$ and use them as diagonalization witnesses. So the e-th requirement will have

$$c_{e+1} - c_e := k \tag{5}$$

numbers available for each of W_1, W_2, from $2c_e + 1$ on. To find a k sufficiently big to guarantee the success of this diagonalization ripple we consider the rectification resources of A below

$$[2(c_{e+1} - 1) + 1 + e] + 1 = 2(c_e + k) + e$$

which bounds the use of any of our e-witnesses. By 4,

$$|A \upharpoonright 2(c_e + k) + e| < c_e + k + \frac{e}{2}$$

and so there can only be less than $c_e + k + \frac{e}{2}$ rectifications to the e-diagonalizations. Since we play with two sets W_1, W_2 (and $\mathcal{N}'_e$ is a disjunction) we have $2k$ witnesses available. Hence it suffices to choose k so that

$$2k \geq c_e + k + \frac{e}{2}$$

i.e. $k \geq c_e + \frac{e}{2}$. By setting $k = c_e + \frac{e}{2}$, $c_0 = 0$ and using 5 we get an appropriate sequence (c_i) (where c_e is the number of witnesses reserved by $\mathcal{N}'_i$, $i < e$) and are able to proceed with the $\mathcal{N}'_e$-strategy.

$\mathcal{N}'_e$-Module.

1. Wait until $\ell(\Phi_e^A, W_i) > 2c_{e+1}+1$ for both $i = 1, 2$.
2. Consider the maximum witness of $\mathcal{N}'_e$ not yet in W_1 or W_2; that is,

$$\max(2\mathbb{N}+1) \cap [2c_e+1, 2c_{e+1}+1) \cap (\overline{W}_1 \cup \overline{W}_2).$$

Put it into W_1 if it is not in already; otherwise enumerated into W_2.

The above strategy requires attention when the condition in the first step is fulfilled. Now the *construction* is straightforward: at stage s run the $\mathcal{Q}$-module *for both* W_i, and the highest priority $\mathcal{N}'_e$-module requiring attention.

Verification. First note that $\mathcal{Q}$ uses only even numbers and each $\mathcal{N}'_e$ only odd ones. So $\mathcal{Q}$ does not have any interaction with the rest of the construction and so $A \leq_{\mathrm{sw}} W_i$ can be derived as explained above; note that the characteristic sequences of W_1, W_2 are identical on the even positions. Moreover there is no interaction between pairs of $\mathcal{N}'$ strategies since their witness intervals are disjoint. Each of these requirements succeeds because of the choice of witness intervals, as explained above. The operation of such a requirement is a sequence of diagonalizations against *one* of the reductions $\Phi_e^A = W_1$, $\Psi_e^A = W_2$, each of which (except the first one) takes place after an A-change below a certain level. When we defined the parameters c_e we showed that these A-changes cannot be as many as the number of witnesses for both W_i. This means that one of these reductions will stop having expansionary stages and thus the $\mathcal{N}'$ strategy will start waiting in step 1 indefinitely, after a certain stage. This is obviously a successful outcome.

Further Remarks. One of the referees has pointed out that theorem 2 can be proved as follows (this approach is closer to the construction of W_1 above). Fix the density of A

$$\alpha = \limsup_{n\to\infty} \frac{|\{m \leq n : A(m) = 1\}|}{n}$$

and a rational approximation d of it with error less than ϵ (say $= \frac{1}{10}$). Then we can effectively choose diagonalization intervals I_k as we did for the construction of W_1, so that after we enumerate axioms on I_k for the computation of A from W, only a small number (relative to the length of I_k) of extra elements can enter A below $\max I_k$ (e.g. $\frac{1}{10}$th of $\max I_k$). Then, choosing $\max I_k$ big enough we can ensure that there is space in I_k for A-coding and enough A-diagonalizations, even for the case when we diagonalize against a functional with use $x + c$ for arbitrary constant c.

Note that roughly speaking the smaller the error ϵ of the approximation to α is, the more non-uniform the proof is. E.g. if we choose $\epsilon \leq \frac{1}{n}$ we can divide the unit interval into n equal parts and consider the centers of these as our rational approximations d. For any given A one of these must be correct and so the corresponding construction is successful. Setting $\epsilon \leq \frac{1}{3}$ suffices: we get three sets W_1, W_2, W_3 one of which satisfies the requirements. It is interesting that there is no obvious way to succeed with less than three attempts.

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